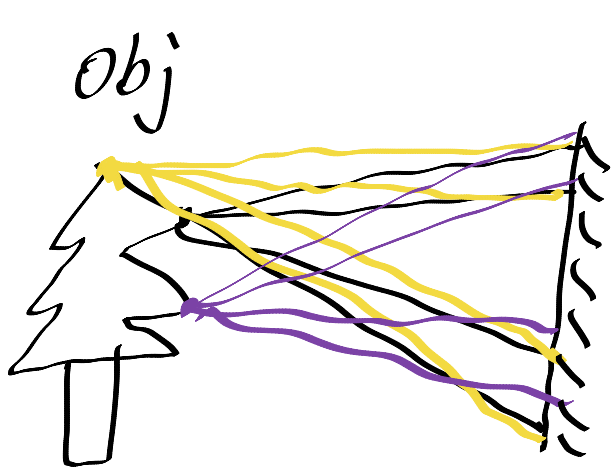


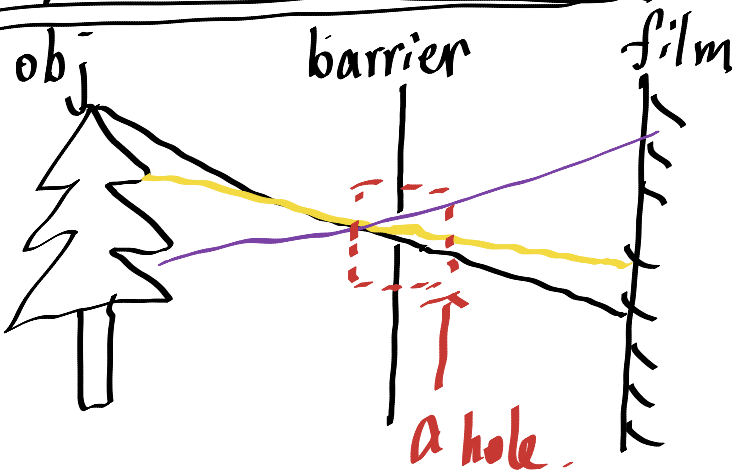
# Lect 2. Perspective Projection

## 1. Direct Projection



If we purely record the object, each pixel on the film will be the super-compose of a lights from the obj.

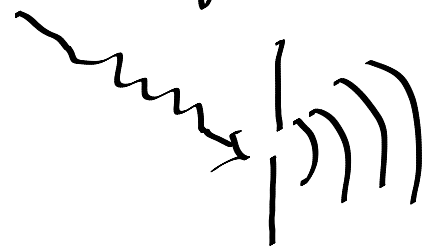
## 2. Pinhole Camera



If the hole on the barrier is small enough, each pixel on the film will only correspond to a certain ray (same direction)

Small hole → clear image  
→ dim image (no enough light)

Too small hole → diffraction  
衍射



# Projection

When do 3D  $\mapsto$  2D projection

Preserved:

Straight line (line  $\rightarrow$  line, line  ~~$\rightarrow$~~  curve)

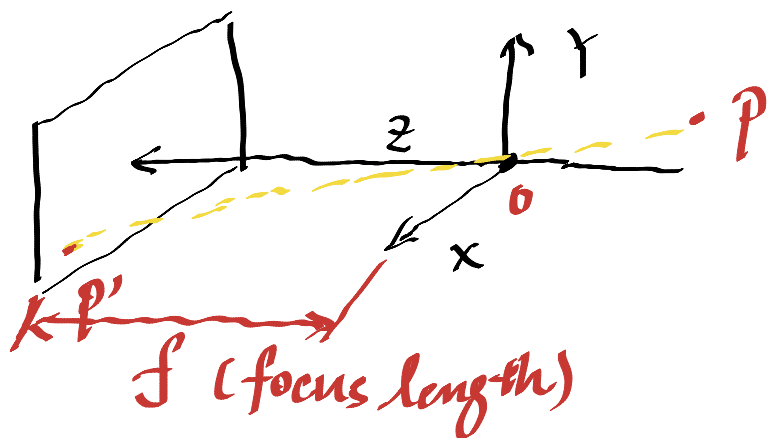
Incidence (obj A touches obj B  ~~$\mapsto$~~  obj A away from obj B)

Not Preserved:

length:  $\square$  3 cm x 3 cm square  $\xrightarrow{\text{maybe}}$  2x2 square  $\square$

length ratio:  $\square$  3 cm x 3 cm square  $\xrightarrow{\text{maybe}}$  2x4 parallelogram  $\square$

angle: right-angle  $\square$   $\xrightarrow{\text{maybe}}$  acute angle  $\square$



If  $O$  is  $(0, 0, 0)$

$P$  is  $(x, y, z)$

$P'$  is  $(\frac{x}{z} \cdot f, \frac{y}{z} \cdot f)$

## Vanishing Point

$\rightarrow$  Parallel lines will intersect at vanishing points

$\rightarrow$  Vanishing point is only corresponding to line direction, or lines of same direction will end at same point.

$\rightarrow$  Only lines parallel to film will stay parallel

# Proof for vanishing point

Start point  $(x_0, y_0, z_0)$

direction  $(u, v, w)$ ,  $t \in \mathbb{R}$

points on the line passing start point can be

$(x_0, y_0, z_0) + t(u, v, w)$  --- denote as point P

Projection of point P onto film as point P'

the coordinate for P' is

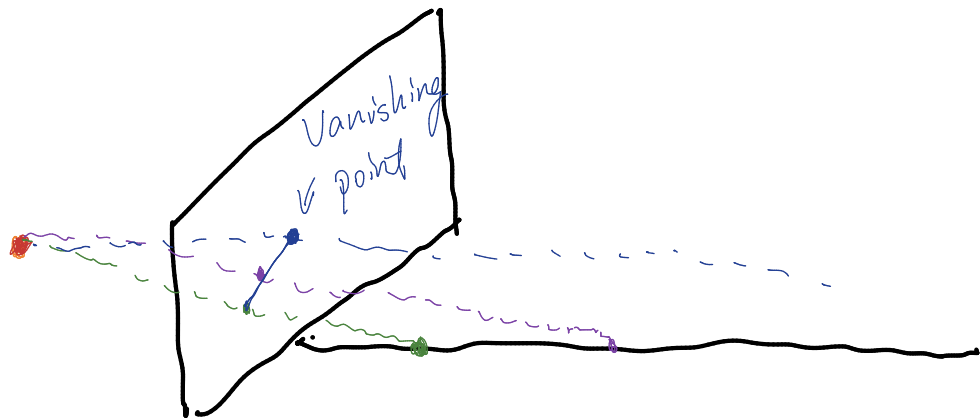
$$\left( f \frac{x_0 + tu}{z_0 + tw}, f \frac{y_0 + tv}{z_0 + tw}, -f \right)$$

$$\lim_{t \rightarrow \infty} f \frac{x_0 + tu}{z_0 + tw} = f \frac{u}{w} \quad \dots \text{have nothing to do with } x_0, z_0$$

Similarly

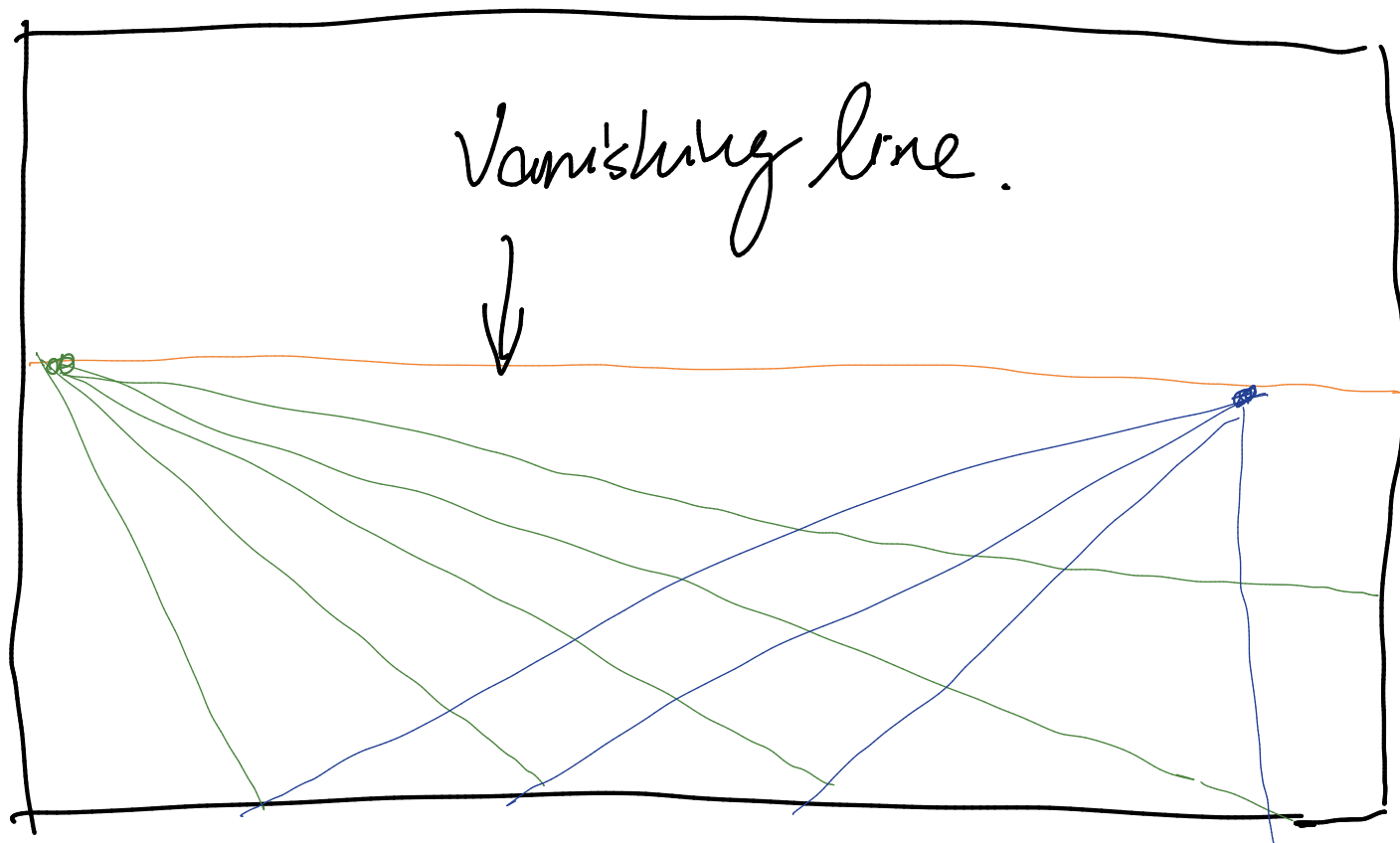
$$\lim_{t \rightarrow \infty} f \frac{y_0 + tv}{z_0 + tw} = f \frac{v}{w}$$

only the direction cares



## Vanishing line.

Plane which is not parallel to film will have vanishing line. Eg. horizon



Orthographic Projection

$$P(x, y, z) \mapsto P'(x, y)$$